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Charged particle motions in the presence of an electromagnetic wave

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Abstract : The charged particle trajectories are evaluated in the presence of constant magnetic field and an electromagnetic wave. The perturbed velocities are derived for the resonant and non-resonant particles. The possible applicabilities of the theory have been also pointed out.

Keywords : Electromagnetic wave, magnetic field, non-resonant and resonant particle trajectories, free-gyration

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Study of motion of charged particles in electromagnetic waves is important in plasma physics. Resonance between waves and charged particles seems to play a considerable part in energy exchange processes in the physics of the magnetosphere of the earth and in controlled thermonuclear experiments [1].

Previously, Leer [2] analysed motions of charged particles in the presence of a plane electromagnetic wave propagating along a constant homogeneous magnetic field. The utility of this theory is the generality of the expressions for the space-time velocity of a particle moving in a general plane wave when the field can be found as a function of the proper time. Liemohn and Duane [1] have also presented the particle motion in the electromagnetic ion cyclotron wave propagation along the constant magnetic field and the applicability of the charged particle trajectories is emphasized by these authors.

Cole [3] has also evaluated the displacement of charged particles in orthogonal electric ($\boldsymbol{E}$) and magnetic field ($\boldsymbol{B}$) in the case where ∇E is non-zero and parallel to $\boldsymbol{E}$. The applications of this theory to structures in the magnetosphere and to plasmas in other planetary and astrophysical setting are pointed out.

Recently, Tiwari and Varma [4,5] and Varma and Tiwari [6] have analysed the motion of charged particles in the presence of electrostatic waves for the drift wave instabilities. Electrostatic ion cyclotron instability was also analysed by the use of charged particle trajectories and energy exchange has been estimated [7]. In this paper, we have considered a plane electromagnetic wave propagating obliquely to the constant magnetic field for the evaluation of charged particle trajectory. The background plasma particles are divided into two groups, the resonant and non-resonant particles [4]. An electromagnetic wave is assumed to start at $t = 0$ when the resonant particles do not participate in the energy exchange process [4,5]. The resonant particles in this case, are those which have the velocities near the phase velocity in the direction along the mean magnetic field [4,5].

The equation of motion for charged particles is considered according to the particle aspect analysis [4–6, 8–10] as :

$$m\frac{d\boldsymbol{v}}{dt} = q\left[\boldsymbol{E}_1 + \frac{1}{c}\,\boldsymbol{v} \times (\boldsymbol{B}_0 + \boldsymbol{B}_1)\right], \tag{1}$$

where m and q are the mass and charge of the particles. Electric field $\boldsymbol{E}_1$ and the magnetic field $\boldsymbol{B}_1$ on the right hand side, are considered as small perturbations due to the electromagnetic waves. Velocity $\boldsymbol{v}$ can be expressed in terms of unperturbed velocity $\boldsymbol{V}$ and the perturbed one $\boldsymbol{u}$. The unperturbed velocity $\boldsymbol{V}$ is given by

$$m\frac{d\boldsymbol{V}}{dt} = q\left[\frac{\boldsymbol{V} \times \boldsymbol{B}_0}{c}\right], \tag{2}$$

which leads to the solution for the trajectory of free gyration as

$$\begin{aligned} x(t) &= \frac{V_\perp}{\Omega}\left[\sin(\theta - \Omega t) - \sin\theta\right] + x_0, \\ y(t) &= -\frac{V_\perp}{\Omega}\left[\cos(\theta - \Omega t) - \cos\theta\right] + y_0, \\ z(t) &= V_\parallel t + z_0, \end{aligned} \tag{3}$$

where $\gamma_0 = (x_0, y_0, z_0)$ is the position of particles at $t = 0$; $\Omega = qB_0/mc$ is the cyclotron frequency, and θ is the initial phase of the velocities. The unperturbed velocity $\boldsymbol{V}$ is taken to be

$$\begin{aligned} V_x(t) &= V_\perp \cos(\theta - \Omega t), \\ V_y(t) &= V_\perp \sin(\theta - \Omega t), \\ V_z(t) &= V_\parallel . \end{aligned} \tag{4}$$

The perturbed velocity $\boldsymbol{u}$ is determined by the following set of equations, considering [11]

$$\boldsymbol{B}_1(\boldsymbol{r},t) = -\frac{\check{}}{\omega}\,\boldsymbol{k} \times \boldsymbol{E}_1(\boldsymbol{r},t) \tag{5}$$

and the wave electric field of the form

$$E_{1y} = \bar{E}_{1y} \cos(k_\perp y + k_\parallel z - \omega t)$$

$$E_{1z} = \bar{E}_{1z} \cos(k_\perp y + k_\parallel z - \omega t) \tag{6}$$

where $k_\perp$ and $k_\parallel$ are the components of wave vector $\boldsymbol{k}$ across and along the static magnetic field $\boldsymbol{B}_0$ which is in z-direction, ω is the frequency of electromagnetic wave considered. The x-component of wave electric field is assumed to be zero. C is the velocity of light.

With the help of eqs. (1), (5) and (6), we get the set of equations for perturbed velocity $\boldsymbol{u}$ as {4,5] :

$$\frac{du_+}{dt} + i\Omega u_+ = \frac{iqE_{\perp y}}{m}\left(1 + \frac{k_z V_z}{\omega}\right) - \frac{iq}{m\omega} k_y E_{\perp z} V_z$$

$$\frac{du_z}{dt} = \frac{qE_{1z}}{m} + \frac{q}{m\omega} V_y \left(k_y E_{1z} - k_z E_{1y}\right) \tag{7}$$

where, $\quad u_+ = u_x + iu_y$

Substituting (3) into (7) and solving these differential equations, we find an oscillatory solution of $\boldsymbol{u}(t)$ for the non-resonant particles. It is necessary to take into account the initial condition $\boldsymbol{u}(t = 0) = 0$ for the resonant particles inferred from the basic assumptions. Thus, by solving the eq. (7) we get the perturbed velocities $\boldsymbol{u}(t)$ as :

$$u_x(t) = \frac{q}{m} \sum_{n=-\infty}^{\infty} J_n(u) \left\{ \bar{E}_{1y}\left(1 + \frac{k_z V_z}{\omega}\right) - \bar{E}_{1z}\frac{k_z V_z}{\omega} \right\}$$

$$\times \left[\frac{\Omega}{a_n} \sin\psi_n + \delta\frac{\sin\left(\psi_n^0 - \Omega t\right)}{2(\Lambda n + \Omega)} - \delta\frac{\sin\left(\psi_n^0 + \Omega t\right)}{2(\Lambda n - \Omega)} \right],$$

$$u_y(t) = \frac{q}{m} \sum_{n=-\infty}^{\infty} J_n(\mu) \left\{ \bar{E}_{1y}\left(1 + \frac{k_z V_z}{\omega}\right) - \bar{E}_{1z}\frac{k_z V_z}{\omega} \right\}$$

$$\times \left[\frac{\Lambda n}{a_n} \cos\psi_n - \delta\frac{\cos\left(\psi_{n_1}^0 - \Omega t\right)}{2(\Lambda_n + \Omega)} - \delta\frac{\cos\left(\psi_n^0 + \Omega t\right)}{2(\Lambda_n - \Omega)} \right],$$

$$u_z(t) = \frac{q}{m} \sum_{n=-\infty}^{\infty} J_n(\mu) \left\{ \bar{E}_{1z} + \left(\frac{V_\perp k_y \bar{E}_{1z}}{\omega} - \frac{V_\perp k_z \bar{E}_{1y}}{\omega}\right) \frac{n}{\mu} \right\}$$

$$\times \frac{1}{\Lambda n} \left[\sin\psi_n - \delta \sin\psi_n^0\right], \tag{8}$$

where $\delta = 0$ for the non-resonant particles and $\delta = 1$ for the resonant ones, and

$$\mu = \frac{k_\perp V_\perp}{\Omega},\ a_n = \left(k_\parallel V_\parallel - \omega - n\Omega\right)^2 - \Omega^2 = \Lambda_n^2 - \Omega^2,$$

$$\psi_n = \Lambda_n t + n\left(\frac{\pi}{2} - \theta\right) + k_\perp y_0 + k_\parallel z_0 - \mu\cos\theta,$$

$$\psi_n^0 = \psi_n(t = 0).$$

$J_n(\mu)$ represents the Bessel function of order n.

Also use was made of the expansions

$$\exp\left[i\mu\cos(\theta - \Omega t)\right] = \sum_{n=-\infty}^{\infty} J_n(\mu)\exp\left[in\left(\frac{\pi}{2} - \theta + \Omega t\right)\right]$$

and $$\sin(\theta - \Omega t)\exp\left[i\mu\cos(\theta - \Omega t)\right] = \sum_{n=-\infty}^{\infty} \frac{n}{\mu} J_n(\mu)\exp\left[in\left(\frac{\pi}{2} - \theta + \Omega t\right)\right]. \quad (9)$$

It is easy to calculate the true trajectory of the particle to first order by first integrating $\mu(t)$ and then adding (3) to it.

If we again substitute (3) into (8) $\boldsymbol{u}(t)$ can be written down in the form of $\boldsymbol{u}(\boldsymbol{r},t)$ as :

$$u_x(\boldsymbol{r},t) = \frac{q}{m}\sum_{n=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} J_n(\mu) J_l(\mu)\left\{\bar{E}_{1y}\left(1 + \frac{k_z V_z}{\omega}\right) - \bar{E}_{1z}\frac{k_z V_z}{\omega}\right\}$$
$$\times\left[\frac{\Omega}{a_n}\sin\chi_{nl} + \delta\frac{\sin(\chi_{nl} - \Lambda_{n-1}t)}{2(\Lambda_n + \Omega)} - \delta\frac{\sin(\chi_{nl} - \Lambda_{n+1}t)}{2(\Lambda_n - \Omega)}\right]$$

$$u_y(\boldsymbol{r},t) = \frac{q}{m}\sum_{n=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} J_n(\mu) J_l(\mu)\left\{\bar{E}_{1y}\left(1 + \frac{k_z V_z}{\omega}\right) - \bar{E}_{1z}\frac{k_z V_z}{\omega}\right\}$$
$$\times\left[\frac{An}{a_n}\cos\chi_{nl} - \delta\frac{\cos(\chi_{nl} - \Lambda_{n-1}t)}{2(\Lambda_n + \Omega)} - \delta\frac{\cos(\chi_{nl} - \Lambda_{n+1}t)}{2(\Lambda_n - \Omega)}\right],$$

$$u_z(\boldsymbol{r},t) = \frac{q}{m}\sum_{n=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} J_n(\mu) J_l(\mu)\left\{\bar{E}_{1z} + \left(\frac{V_\perp k_y \bar{E}_{1z}}{\omega} - \frac{V_\perp k_z E_{1y}}{\omega}\right)\frac{n}{\mu}\right\}$$
$$\times\frac{1}{\Lambda_n}\left[\sin\chi_{nl} - \delta\sin(\chi_{nl} - \Lambda_n t\right], \quad (10)$$

where $\delta = 0$ for the non-resonant and $\delta = 1$ for the resonant particles and

$$\chi_{nl} = k_\perp y + k_\parallel z - \omega t + (n - l)\left(\frac{\pi}{2} + \theta - \Omega t\right).$$

These equations represent the motion of charged particle in electromagnetic field along the x, y and z directions. The trajectories have wide applications in plasma heating processes, confinement devices and the space plasmas [1,6,9]. In the magnetospheric plasma and in the auroral acceleration region, current and heating of the charged particles can be estimated through the use of these trajectories. The charged particle trajectories are also used in the analysis of plasma instabilities by particle aspect theories [4,6].

The interaction between charged particles and electromagnetic waves propagating in the presence of a static magnetic field is an important problem in classical plasma physics owing to possible energy exchange at the cyclotron resonance [12]. The motion of ions in such fields is known to be described by elliptic functions [1] but the physics is obscured by the complicated expressions. The properties of the particle trajectories are much more transparent in the perturbation expansion solution derived here. For small wave amplitudes, the solutions are valid for an extended time and the resonance effects are clearly evident. The initial phase between the particles and wave fields emerges as an important condition on the motion. Potential applications of the results in magnetospheric physics and plasma fusion devices are of importance [1]. These charged particle trajectories may be further utilized to explain the energy exchange and heating of the plasma particles due to the plasma waves and in the process of computer simulation [13].

A good understanding of particle orbits is very important for predicting the plasma confinement, high energy particle loss and heating efficiency in fusion devices [14]. To study particle orbits several techniques of orbit calculations have been discussed. A typical method is to study the particle trajectories of many particles on the basis of guiding center drift equations. Another method is to use the adiabatic invariants [15,16]. The former method requires in general, more computations than the latter. In tenuous or high temperature plasmas where collisional encounters between the constituent charged particles are rare, an understanding of plasma phenomena requires a knowledge of the individual particle trajectories in the self consistent electromagnetic fields. Many phenomena can, however, be investigated and explained on the basis of elementary orbit theory in given externally imposed fields. The study of motion of charged particles provides an natural bridge between the macroscopic fluid theory and the microscopic Vlasov theory. In the limit of a cold plasma, the fluid elements follow the trajectory of a charged particle, since all the particles move in concert. However, in a warm plasma, the velocity and configuration space motions of individual particles are ensemble – averaged to give the statistical kinetic equation describing plasma properties. In either case, the knowledge of motion of individual particles is helpful in the various situations that arise in plasma physics.

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References

[1] H B Liemohn and B H Duane *Am. J. Phys* **44** 100 (1976)

[2] E Leer *Phys. Fluids* **12** 2206 (1969)

[3] K D Cole *Planet. Space Sci.* **24** 515 (1976)

[4] M S Tiwari and P Varma *J Plasma Phys.* **46** 49 (1991)

[5] M S Tiwari and P Varma *Planet Space Sci* **41** 207 (1993)

[6] P Varma and M S Tiwari *Indian J Pure Appl Phys.* **31** 616 (1993)

[7] V K Bajaj and M S Tiwari *Indian J Phys.* **67B** 343 (1993)

[8] P Varma and M S Tiwari *Phys. Scripta* **44** 296 (1991)

[9] P Varma and M S Tiwari *Phys. Scripta* **45** 275 (1992)

[10] P Varma and M S Tiwari *Indian J. Pure Appl Phys* **30** 1 (1992)

[11] Y Terashima *Progr. Theo. Phys* **37** 775 (1967)

[12] I Roth and M Temerin *IEEE Trans Plasma Sci* **14** 910 (1986)

[13] H Matsumoto *Phys. Fluids* **20** 2093 (1977)

[14] H Sanuki, J Todoroki and T Kamimura *Research Report No NIFS-9, National Institute for Fusion Science, Japan* (1990)

[15] T G Northrop *Ann Phys* **15** 79 (1961)

[16] T G Northrop The Adiabatic Motion of Charged Particles (New York : Willey) (1963)